BROJNI REDOVI

106. Ispitati konvergenciju redova

a) $\sum_{n=1}^{\infty} \frac{\sqrt[3]{n^3+1}}{n}$

→ opšti član je $a_n = \frac{\sqrt[3]{n^3+1}}{n}$

JORDI LABANDA

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{\sqrt[3]{n^3\left(1+\frac{1}{n^3}\right)}}{n} = \lim_{n\to\infty} \frac{n\sqrt[3]{1+\frac{1}{n^3}}}{n} = 1 \neq 0$$

$\rightarrow$ red $\sum_{n=1}^{\infty} a_n$ ne konvergira, tj. divergira.

$\rightarrow$ Da bi red konvergirao $\rightarrow$ njegov opšti član mora da teži nuli

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b) $\sum_{n=1}^{\infty} \frac{n+1}{2n+3}$

$\rightarrow$ opšti član $a_n = \frac{n+1}{2n+3}$

$\lim_{n\to\infty} \frac{n+1}{2n+3} = \frac{1}{2} \neq 0 \Rightarrow$ red $\sum_{n=1}^{\infty} \frac{n+1}{2n+3}$ divergira

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c) $\sum_{n=1}^{\infty} \ln \frac{n}{n+1}$ $\longrightarrow$ niz parcijalnih suma ne konvergira $\rightarrow$ ne konvergira ni brojni red

$a_n = \ln \frac{n}{n+1} = \ln n - \ln(n+1)$

$S_n = a_1 + a_2 + \dots + a_n = \ln 1 - \cancel{\ln 2} + \cancel{\ln 2} - \cancel{\ln 3} + \dots + \cancel{\ln n} - \ln(n+1)$

$= \underbrace{\ln 1}_{0} - \ln(n+1) = -\ln(n+1)$

$\lim_{n\to\infty} S_n = -\lim_{n\to\infty} \ln(n+1) = -\infty \longrightarrow$

Niz $(S_n)_{n\in\mathbb{N}}$ ne konvergira, pa ne konvergira ni red $\sum_{n=1}^{\infty} a_n$

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d) $\sum_{n=1}^{\infty} \ln\left(\frac{n^2+1}{2n^2+1}\right)$ $a_n = \ln \frac{n^2+1}{2n^2+1}$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \ln\left(\frac{n^2\left(1+\frac{1}{n^2}\right)}{n^2\left(2+\frac{1}{n^2}\right)}\right) = \ln\frac{1}{2} \neq 0$$

$\Rightarrow$ red $\sum_{n=1}^{\infty} a_n$ ne konvergira

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(107) Naći sumu reda $\sum_{n=1}^{\infty} a_n$, gdje je ~~a_n~~

$a_n = \sqrt{n+2} - 2\sqrt{n+1} + \sqrt{n}$

$\Rightarrow$ posmatra se niz parcijalnih suma

$S_n = a_1 + a_2 + \cdots + a_n$

$$S_n = (\sqrt{3} - 2\sqrt{2} + \sqrt{1}) + (\sqrt{4} - 2\sqrt{3} + \sqrt{2}) + (\sqrt{5} - 2\sqrt{4} + \sqrt{3}) + \cdots$$

$$+ (\sqrt{n} - 2\sqrt{n-1} + \sqrt{n-2}) + (\sqrt{n+1} - 2\sqrt{n} + \sqrt{n-1}) +$$

$$+ \sqrt{n+2} - 2\sqrt{n+1} + \sqrt{n}$$

$$S_n = 1 - \sqrt{2} - \sqrt{n+1} + \sqrt{n+2}$$

racionališemo

gr. vr. niza parc. suma

$$\Rightarrow \lim_{n\to\infty} S_n = \lim_{n\to\infty}\left(1 - \sqrt{2} + \left(\left(\sqrt{n+2} - \sqrt{n+1}\right) \cdot \frac{\sqrt{n+2}+\sqrt{n+1}}{\sqrt{n+2}+\sqrt{n+1}}\right)\right) =$$

$$= \lim_{n\to\infty}\left(1 - \frac{1}{\sqrt{n+2}+\sqrt{n+1}}\right) = 1 - \sqrt{2}$$

$\Rightarrow$ NPS S_n konvergira pa konvergira i red $\sum_{n=1}^{\infty} a_n$

(108) Naći sumu reda $\sum_{n=1}^{\infty} \frac{1}{(3n-2)(3n+1)}$

$$a_n = \frac{1}{(3n-2)(3n+1)} = \frac{A}{3n-2} + \frac{B}{3n+1}$$

$$A = \frac{1}{3}; \quad B = -\frac{1}{3}$$

$$a_n = \frac{1}{3}\left(\frac{1}{3n-2} - \frac{1}{3n+1}\right)$$

$$\rightarrow S_n = a_1 + a_2 + \dots + a_n =$$

$$= \frac{1}{3}\left(\frac{1}{1} - \cancel{\frac{1}{4}} + \cancel{\frac{1}{4}} - \cancel{\frac{1}{7}} + \dots + \cancel{\frac{1}{3n-2}} - \frac{1}{3n+1}\right) =$$

$$= \frac{1}{3}\left(1 - \frac{1}{3n+1}\right)$$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{1}{3}\left(1 - \frac{1}{3n+1}\right) = \frac{1}{3}$$

$\rightarrow$ Niz parcijalnih suma (S_n) konvergira $\rightarrow$

$\rightarrow$ konvergira i red $\sum_{n=1}^{\infty} a_n$

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(109) Naći sumu reda $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \dots$

$$\sum_{n=1}^{\infty} (-1)^{n-1} \cdot \frac{1}{2^{n-1}} = \sum_{n=1}^{\infty} \left(-\frac{1}{2}\right)^{n-1}$$

$$\rightarrow a_n = \left(-\frac{1}{2}\right)^{n-1}; \quad S_n = a_1 + a_2 + \dots + a_n$$

$$S_n = 1 - \frac{1}{2} + \frac{1}{4} + \dots + \left(-\frac{1}{2}\right)^{n-1} = 1 \cdot \frac{1-\left(-\frac{1}{2}\right)^n}{1-\left(-\frac{1}{2}\right)} =$$

$$= \frac{2}{3}\left(1 - \left(-\frac{1}{2}\right)^n\right)$$

$$\lim_{n\to\infty} \frac{2}{3}\left(1 - \underbrace{\left(-\frac{1}{2}\right)^n}_{0}\right) = \frac{2}{3} \rightarrow \text{niz } S_n \text{ konvergira}$$

pa konvergira i red $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{2^{n-1}}$

(110) Ispitati konvergenciju i naći sumu reda $\sum_{n=1}^{\infty} \frac{e^n}{3^{n-1}}$

$$a_n = \frac{e^n}{3^{n-1}} = e\left(\frac{e}{3}\right)^{n-1}$$

$\rightarrow$ Ovo je geometrijski red $\rightarrow e, e\frac{e}{3}, e\left(\frac{e}{3}\right)^2, \dots$

$\rightarrow q = \frac{e}{3} \rightarrow |q| < 1 \rightarrow$ Dati geometrijski red konvergira.

(111) Ispitati konvergenciju reda $\sum_{n=1}^{\infty}\left(\frac{1}{e^n} + \frac{1}{n(n+1)}\right) =$

$$= \sum_{n=1}^{\infty} \frac{1}{e^n} + \sum_{n=1}^{\infty} \frac{1}{n(n+1)}$$

$\rightarrow$ opšti član $a_n = \frac{1}{e^n} + \frac{1}{n(n+1)}$

$\rightarrow$ Neka je $b_n = \frac{1}{e^n}$ i $c_n = \frac{1}{n(n+1)}$

$\sum_{n=1}^{\infty} \frac{1}{e^n}$ konvergira kao geometrijski red kod kojeg je $q = \frac{1}{e} < 1$

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$$

$$c_n = -\frac{1}{n+1} + \frac{1}{n} \rightarrow \frac{1}{n} - \frac{1}{n+1}$$

$$S_n = c_1 + c_2 + \dots + c_n = 1 - \cancel{\frac{1}{2}} + \cancel{\frac{1}{2}} - \cancel{\frac{1}{3}} + \dots + \cancel{\frac{1}{n}} - \frac{1}{n+1} =$$

$$= 1 - \frac{1}{n+1}$$

$\lim_{n\to\infty} S_n \left(1 - \cancel{\frac{1}{n+1}}\right) = 1 \rightarrow$ niz S_n konv $\rightarrow$

$\rightarrow$ konv. i red $\sum_{n=1}^{\infty} c_n$